CALCULUS

Limits
Common Derivatives
\[
\lim_{x \to a} f(x) = L \quad \phi 1
\]
\[
\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} f(x) = L 
\]
\[
\lim_{x \to a} f(x) \bigg/ \lim_{x \to a} g(x) \quad \phi 1
\]
\[
\lim_{x \to a} f(x) \bigg/ \lim_{x \to a} g(x) \quad \text{Does Not Exist}
\]

L'Hopital's Rule
\[
\text{If } \lim_{x \to a} f(x) = 0 \quad \text{or} \quad \lim_{x \to a} g(x) = \pm \infty\quad \text{then},
\]
\[
\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} \quad a \text{ is a number, } \pm \text{ or } -
\]

Derivatives
Definition and Notation
\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
\]

Basic Properties and Formulas
\[(f \cdot g)' = f' \cdot g + f \cdot g' \quad - \text{Product Rule}
\]
\[
\frac{d}{dx} \left( \frac{f}{g} \right) = \frac{f' \cdot g - f \cdot g'}{g^2} \quad - \text{Quotient Rule}
\]
\[
\frac{d}{dx} \left( x^n \right) = n \cdot x^{n-1} \quad - \text{Power Rule}
\]
\[
\frac{d}{dx} \left( f \left( g(x) \right) \right) = f' \left( g(x) \right) \cdot g'(x)
\]

This is the Chain Rule
\[
\frac{d}{dx} \left( \csc x \right) = -\csc x \cot x
\]
\[
\frac{d}{dx} \left( \sin x \right) = \cos x
\]
\[
\frac{d}{dx} \left( \cot x \right) = -\csc^2 x
\]
\[
\frac{d}{dx} \left( \cos x \right) = -\sin x
\]
\[
\frac{d}{dx} \left( \sin^{-1} x \right) = \frac{1}{\sqrt{1-x^2}}
\]
\[
\frac{d}{dx} \left( \sec x \right) = \sec x \tan x
\]
\[
\frac{d}{dx} \left( \tan x \right) = \sec^2 x
\]
\[
\frac{d}{dx} \left( \cos^{-1} x \right) = -\frac{1}{\sqrt{1-x^2}}
\]
\[
\frac{d}{dx} \left( \tan^{-1} x \right) = \frac{1}{1 + x^2}
\]

Increasing/Decreasing
Concave Up/Concave Down
Critical Points
\[x = c \text{ is a critical point of } f(x) \text{ provided either}
1. \ f'(c) = 0 \text{ or } 2. \ f'(c) \text{ doesn't exist.}

Increasing/Decreasing
1. If \ f'(x) > 0 \text{ for all } x \text{ in an interval } I \text{ then}
\ f(x) \text{ is increasing on the interval } I.
2. If \ f'(x) < 0 \text{ for all } x \text{ in an interval } I \text{ then}
\ f(x) \text{ is decreasing on the interval } I.
3. If \ f'(x) = 0 \text{ for all } x \text{ in an interval } I \text{ then}
\ f(x) \text{ is constant on the interval } I.

Concave Up/Concave Down
1. If \ f''(x) > 0 \text{ for all } x \text{ in an interval } I \text{ then}
\ f(x) \text{ is concave up on the interval } I.
2. If \ f''(x) < 0 \text{ for all } x \text{ in an interval } I \text{ then}
\ f(x) \text{ is concave down on the interval } I.

Inflection Points
\[x = c \text{ is an inflection point of } f(x) \text{ if the concavity changes at } x = c.

The information for this handout was compiled from the following sources:
Calculus Formulas

1st Derivative Test
If \( x = c \) is a critical point of \( f(x) \) then \( x = c \) is
1. a rel. max. of \( f(x) \) if \( f'(x) > 0 \) to the left
   of \( x = c \) and \( f'(x) < 0 \) to the right of \( x = c \).
2. a rel. min. of \( f(x) \) if \( f'(x) < 0 \) to the left
   of \( x = c \) and \( f'(x) > 0 \) to the right of \( x = c \).
3. not a relative extremum of \( f(x) \) if \( f'(x) \) is
   the same sign on both sides of \( x = c \).

2nd Derivative Test
If \( x = c \) is a critical point of \( f(x) \) such that
\( f''(c) = 0 \) then \( x = c \)
1. is a relative maximum of \( f(x) \) if \( f''(c) < 0 \).
2. is a relative minimum of \( f(x) \) if \( f''(c) > 0 \).
3. may be a relative maximum, relative minimum, or neither if \( f''(c) = 0 \).

Fundamental Theorem of Calculus
Part I: \( \frac{d}{dx} \left[ \int_a^b f(t) \, dt \right] = f(x) \)
Part II: \( \int_a^b f(x) \, dx = F(b) - F(a) \)

Common Integrals
\[ \sqrt{k} \, dx = k \, x + c \]
\[ x^n \, dx = \frac{1}{n+1} x^{n+1} + c, \quad n \neq -1 \]
\[ \sqrt{x} \, dx = \ln|x| + c \]
\[ \frac{1}{ax+b} \, dx = \frac{1}{a} \ln|ax+b| + c \]
\[ \ln u \, du = u \ln(u) - u + c \]
\[ e^u \, du = e^u + c \]
\[ \cos u \, du = \sin u + c \]
\[ \sin u \, du = -\cos u + c \]
\[ \sec^2 u \, du = \tan u + c \]
\[ \sec u \tan u \, du = \sec u + c \]
\[ \csc u \cot u \, du = -\csc u + c \]
\[ \csc^2 u \, du = -\cot u + c \]

Integration by Parts:
\[ \int u \, dv - v \, du = \int (u \, dv - v \, du) \]

Products and (some) Quotients of Trig Functions
For \( \int \sin^n x \cos^m x \, dx \) we have the following:
1. \( n \) odd. Strip 1 sine out and convert rest to
   cosines using \( \sin^2 x = 1 - \cos^2 x \), then use
   the substitution \( u = \cos x \).
2. \( m \) odd. Strip 1 cosine out and convert rest
   to sines using \( \cos^2 x = 1 - \sin^2 x \), then use
   the substitution \( u = \sin x \).
3. \( n \) and \( m \) both odd. Use either 1. or 2.
4. \( n \) and \( m \) both even. Use double angle
   and/or half angle formulas to reduce the
   integral into a form that can be integrated.

For \( \int \tan^n x \sec^m x \, dx \) we have the following:
1. \( n \) odd. Strip 1 tangent and 1 secant out and
   convert the rest to secants using
   \( \tan^2 x = \sec^2 x - 1 \), then use the substitution
   \( u = \sec x \).
2. \( m \) even. Strip 2 secants out and convert rest
   to tangents using \( \sec^2 x = 1 + \tan^2 x \), then
   use the substitution \( u = \tan x \).
3. \( n \) odd and \( m \) odd. Use either 1. or 2.
4. \( n \) even and \( m \) odd. Each integral will be
   dealt with differently.

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Trig Substitutions:
\[
\begin{align*}
\sqrt{a^2 - b^2} x^2 & \quad x = \frac{a}{b} \sin \theta \\
\cos^2 \theta &= 1 - \sin^2 \theta \\
\sqrt{b^2 x^2 - a^2} & \quad x = \frac{a}{b} \sec \theta \\
\tan^2 \theta &= \sec^2 \theta - 1 \\
\sqrt{a^2 + b^2} x^2 & \quad x = \frac{a}{b} \tan \theta \\
\sec^2 \theta &= 1 + \tan^2 \theta
\end{align*}
\]

Partial Fractions:
Factor in \(Q(x)\) & Term in P.F.D
\[
\begin{align*}
ax + b & \quad \frac{A}{ax + b} \\
ax^2 + bx + c & \quad \frac{Ax + B}{ax^2 + bx + c}
\end{align*}
\]

Area Between Curves:
\[
\begin{align*}
& y = f(x) & A &= \int_b^d [\text{upper function}] - [\text{lower function}] \, dx \\
& x = f(y) & A &= \int_a^b [\text{right function}] - [\text{left function}] \, dy
\end{align*}
\]

Volumes of Revolution:
\[
V = \sqrt[3]{A(x)} \, dx \quad \text{and} \quad V = \sqrt[3]{A(y)} \, dy
\]

Rings
\[
A = \pi \left( \left( \text{outer radius} \right)^2 - \left( \text{inner radius} \right)^2 \right)
\]

Cylinders
\[
A = 2\pi \left( \text{radius} \right) \left( \text{width} / \text{height} \right)
\]

Work:
\[
W = \int_a^b F(x) \, dx
\]

Average Function Value:
\[
\bar{f} = \frac{1}{b-a} \int_a^b f(x) \, dx
\]

Arc Length & Surface Area:
\[
SA_a^b = \sqrt[3]{2\pi y \, ds} \quad \text{(rotate about x-axis)} \\
SA_a^b = \sqrt[3]{2\pi x \, ds} \quad \text{(rotate about y-axis)}
\]

Improper Integral

Infinite Limit
1. \[\int_a^b f(x) \, dx = \lim_{\xi \to a^+} \int_a^\xi f(x) \, dx\]
2. \[\int_a^b f(x) \, dx = \lim_{\eta \to b^-} \int_\eta^b f(x) \, dx\]
3. \[\int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx\]

Discontinuous Integrand
1. Discont. at \(a\): \[\int_a^b f(x) \, dx = \lim_{\xi \to a^+} \int_a^\xi f(x) \, dx\]
2. Discont. at \(b\): \[\int_a^b f(x) \, dx = \lim_{\eta \to b^-} \int_\eta^b f(x) \, dx\]
3. Discontinuity at \(a < c < b\):
\[\int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx\]

Comparison Test for Improper Integrals:
If \[\int_a^b f(x) \, dx \quad \text{and} \quad \int_a^b g(x) \, dx\]
then,
1. If \[\int_a^b f(x) \, dx \quad \text{conv.} \quad \text{then} \quad \int_a^b g(x) \, dx \quad \text{conv.}\]
2. If \[\int_a^b g(x) \, dx \quad \text{divg.} \quad \text{then} \quad \int_a^b f(x) \, dx \quad \text{divg.}\]

Useful fact: If \(a > 0\) then
\[\int_a^b \frac{1}{x^p} \, dx \quad \text{converges if} \quad p > 1 \quad \text{and diverges for} \quad p \leq 1\]

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